A Mathematical Contribution to the Analysis of Moisture Migration in Power Transformer Oil-Paper Insulation Systems


Abstract — This paper proposes mathematical models of moisture in paper-oil insulation systems for use in power transformers. This work is important because it helps shed light on the phenomenon of moisture migration in power transformer insulation systems, and also serves as a tool to underpin decisions by maintenance engineers in response to the occurrence of phenomena that may impair the performance of transformers. The methodology involves the use of computational intelligence tools and the finite difference method. The paper concludes with the description of a case study.

Index Terms — Finite difference method, genetic programming, maintenance, moisture, transformers

I. INTRODUCTION

Power transformers stand out among the various devices that make up Brazil’s National Interconnected System (SIN). Given the high cost of these devices, and the fact that their removal from operation implies the shutdown of large loads, it is very important to monitor their service life [1].

Several parameters in power transformers require monitoring. High concentrations of water can have harmful effects on transformers, such as aging of their insulation system, decreased dielectric strength of the oil, and possible formation of bubbles in the insulation system, increasing risk of fault [2].

In this regard, the purpose of this paper is to present new approaches to the study of moisture in power transformers that differ from the traditional techniques currently in use.

The differential of this work is the use of finite differences method combined with a simpler equation, when confronted with other researches [3-5]. In addition, the focus of the model presented here is its application to transformer temperature curves that consider the winding temperatures along daily full cycles, inferred accurately based on actual measurements of load profiles, tap curves, ambient temperature curves, and status of forced ventilation. This analysis differs from other works, as in [3] and [4], which considers ideal winding temperature curves.

Thus, the main contributions of this study are:

a) The development of an equation that models graphics and curves presented in the literature to underpin decision-making.

b) Proposal of a numerical model to analyze the dynamic behavior of water migration in paper-oil insulation systems, using finite difference method.

II. MOISTURE IN POWER TRANSFORMERS

According to Griffin et al. [6], studies concerning variations in moisture profiles can be divided into steady state and transient state.

A. Steady State – Fabre-Pichon and Oommen curves

The elements that make up the inside of a power transformer tank can be divided into three components: the windings, which carry the electric current; kraft paper, which serves to electrically insulate the coil windings from each other and from the other conductive parts; and the insulating oil, which helps to insulate the active part of the transformer and cools the equipment [6].

Among these three components, water should be present only in the insulating oil and cellulosic material, particularly in paper, because the surfaces of the winding conductors are considered to be watertight. However, the windings play an important role in the study of moisture in transformers, since the temperatures reached by the windings influence the concentrations of water in the oil and paper. In general, when a transformer’s load increases, the temperatures of its windings also increase, and so does the solubility of water in the oil [6].
Insulating mineral oil, a liquid by-product of the distillation of petroleum, has polar portions in its chemical chain that enable it to interact with water molecules. Kraft paper is hygroscopic, and therefore absorbs much more water than oil. There are several curves that model the correlation between the amount of water in oil and in paper, using as an intermediate variable the various temperature curves of the oil sample. This paper will discuss two sets of curves: the one created by Fabre and Pichon, which is displayed in Fig. 1 [7], and the one created by Oommen, illustrated in Fig. 2 [2].

B. Transient State

The transient process of water migration into the transformer is a highly complex phenomenon that depends on the geometry of the inner part of the transformer, the temperature gradient, and other variables [6]. However, a preliminary analysis can be performed by applying Fick’s law [8], as shown in Eq. (1).

\[
\frac{\partial C}{\partial t} = D \cdot \nabla^2 C = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right)
\]

In Eq. (1), \( C \) is the concentration of water in the paper, \( t \) is the time variable, and \( x, y \) and \( z \) are spatial variables. The magnitude \( D \) is the diffusion coefficient, a parameter that indicates the speed of water migration to the cellulose. The symbol \( \nabla^2 \) is the Laplacian operator.

III. FABRE-PICHON AND OOMMEN CURVES USING GENETIC PROGRAMMING

There are several computational techniques for solving problems [9]. These techniques include metaheuristics, which stand out for their enormous arsenal of troubleshooting methods and techniques. A heuristic method is a procedure that will probably find an excellent viable solution, but not necessarily an optimal solution for the specific problem in question [9].

One of the advantages of heuristic algorithms over classic problem-solving methods is that the system’s behavior does not have to be known in order to solve it. However, the disadvantage of heuristics is the fact that the point found, which is considered optimal by the algorithm, may not correspond to the global optimum of the problem. Hence, the use of this tool is recommended for problems that are not solved by traditional methods.

A. Evolutionary Algorithms and Genetic Programming

Metaheuristics can be divided into several categories, including that of evolutionary algorithms, which use the concepts of biology to solve computational problems. These concepts include Charles Darwin’s theory of evolution and the theory of genetics first studied by Gregor Mendel. Thus, the interaction that occurs between individuals of the same species in nature serves as an inspiration for interactions among a number of proposed solutions to the problem. Analogous to nature, these proposed solutions are guided by the concepts of reproduction, evolution and genetics [10].

The Genetic Programming algorithm is an example of evolutionary techniques, whose main characteristic is the fact that the individuals that make up the population are known data structures such as trees, and a reading of each tree provides information about each individual. The trees can be built using constants and/or functions. The genotype of each individual is therefore a mathematical equation.

B. Analyzed curves

The curves were analyzed using a Genetic Programming tool, and points were collected with the aid of graphic design software.

The data were then debugged and analyzed and the results are shown in Figs. 3 and 4 for the Fabre-Pichon and Oommen curves, respectively.
IV. ONE-DIMENSIONAL MATHEMATICAL MODEL

A. The model

A one-dimensional model was developed to represent (1) in the paper-oil system. The main tool used in this model is the finite difference method [11]. By applying this method to Fick’s equation [8], and considering only the x-axis, one obtains (2):

\[ \frac{C_i^{i+1} - C_i^i}{\Delta t} = D \left( \frac{C_i^{i+1} - 2C_i^i + C_i^{i-1}}{\Delta x^2} \right) \]

(2)

where \( C_i^m \) and \( C_i^{m+1} \) are the concentrations of water in element \( m \) of the paper and in the current and subsequent iterations, respectively. \( C_i^m+1 \) and \( C_i^m-1 \) indicate the concentrations of water in the paper, in the current iteration and in positions \( m+1 \) and \( m-1 \), respectively. The variables \( D \), \( \Delta t \) and \( \Delta x^2 \) represent, respectively, the diffusion coefficient, the variation in time, and the square of the variation in space.

Figure 5 shows a representative diagram of the 1D model presented in this paper and modeled by (2).

B. Case Study

After modeling the paper-oil system, the one-dimensional model was implemented using software. To verify the behavior of the mathematical model presented in the Brazilian standard NBR 5416 [7], a case study was conducted using data from a 33.3 MVA power transformer in operation, typically used in power distribution networks. The input parameters for the software are the transformer construction data and an oil sample, which are described in Table IV. Another input variable is the transformer load curve shown in Fig. 6, where the x-axis represents time and the y-axis indicates the temperature at the hottest point in the winding. We used a typical curve of a working day for this transformer, where it is common to reach 100% load at the load end.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fabre-Pichon</th>
<th>Oommen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>Fitness</td>
<td>0.27105</td>
<td>0.25773</td>
</tr>
</tbody>
</table>

Other models for curves, obtained by polynomial fitting and interpolation of points, are presented by Sousa et al. [1]. A second test was performed to compare the quality of fit of the curves shown in Figs. 1 and 2, using Genetic Programming. In this test, several points were selected (with the coordinates: concentration of water in paper, temperature of the oil sample), and the outputs of the three fits (concentration of water in paper) were compared. The results comparing the fittings of the Fabre-Pichon and Oommen curves are described, respectively, in Tables II and III.

### Table II

<table>
<thead>
<tr>
<th>Type of Fit</th>
<th>Fabre-Pichon</th>
<th>Polynomial</th>
<th>Interpolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>7.6314</td>
<td>6.1072</td>
<td>1.0206</td>
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### Table III

<table>
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<tr>
<th>Type of Fit</th>
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<th>Polynomial</th>
<th>Interpolation</th>
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<tr>
<td>Fitness</td>
<td>1.6043</td>
<td>3.3010</td>
<td>0.1128</td>
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</table>

### Table IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Voltage ratios</td>
<td>138/13.8 kV</td>
</tr>
<tr>
<td>Power</td>
<td>20/26.6/33.3 MVA</td>
</tr>
<tr>
<td>Mass of Oil</td>
<td>11000 kg</td>
</tr>
<tr>
<td>Mass of Paper</td>
<td>400 kg</td>
</tr>
<tr>
<td>Temperature of the Sample</td>
<td>40 °C</td>
</tr>
<tr>
<td>Concentration of Water in the Oil Sample</td>
<td>20 ppm</td>
</tr>
</tbody>
</table>
The software developed here returns output graphics, indicating the behavior of moisture in the paper-oil system. Figure 7 shows variation in the concentration of water in the paper throughout the distance along its thickness and over time, in the form of a color chart.

The blue curve in Fig. 8 illustrates the variation in the concentration of water in the oil, in ppm. The green curve shows the relative water saturation level in the oil, in %. This quantity mathematically represents the ratio of water content in a sample to its saturation water content, and is calculated by (3), where $SR$ is the relative saturation of the oil [%], $C_{oil}$ is the concentration of water in the oil, and $C_{oil,sat}$ is the concentration of water saturation in the oil [6].

$$SR = \frac{C_{oil}}{C_{oil,sat}} \times 100\%$$  \hspace{1cm} (3)

The concentration of water saturation in oil is calculated according to (4) [6].

$$C_{oil,sat} = A \cdot e^{-\frac{B}{\theta+273}}$$  \hspace{1cm} (4)

In (4), constants $A$ and $B$ change according to the type of insulating oil, as well as its state of aging.

Figure 9 shows the variation of the water mass in paper (blue curve) and oil (green curve) over time.

It should be noted that variations in water concentrations at the paper-oil interface are of particular interest owing to the risk of failure of the insulation when the transformer is subjected to severe operating conditions, such as abrupt load increases or drops, or special loading conditions. This interest is due to the probability of blistering or free water occurring in this region, causing decay of dielectric strength and the emergence and/or development of partial discharges, which may damage the insulation system and thus cause the transformer to fail.
V. CONCLUSIONS

Fabre-Pichon and Oommen curve fitting by Genetic Programming is efficient, with errors in the same order of magnitude as polynomial fit errors. Whenever possible, interpolation fitting is used to obtain more reliable results.

An analysis of Figs. 5 to 8 confirms that the temperature inside the transformer and moisture in the paper are strongly correlated. Therefore, raising the winding temperature causes the water concentration in the oil to increase, which in turn decreases the concentration of water in the paper. This phenomenon is caused by the migration of moisture from the paper to the oil due to the increase in the concentration of water saturation in insulating mineral oils. Moisture variations in the inner layers of paper follow the moisture variations in the paper at the paper-oil surface. Thus, the low diffusion coefficient clearly indicates that the depth of penetration of water in the paper is small, i.e., in the order of a few millimeters. As mentioned earlier, these variations in water concentration at the paper-oil interface are of particular interest because of the risk of failure of the insulation system when the transformer is subjected to severe operating conditions, which increase the probability of equipment failure.

Therefore, it can also be concluded that the model developed in this study contributes to the qualitative and quantitative analysis of existing studies.

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REFERENCES


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